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LETTER TO THE EDITOR

Dispersive part of the box diagram amplitude

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Abstract. A method to evaluate the dispersive part of the box diagram amplitude is given. The dispersive part thus obtained is shown to be smaller than the absorptive part in the energy region from an incident meson momentum of 1·5 GeV/*c* to a momentum of 4 GeV/*c*.

Recently we have calculated (Agarwal *et al* 1971, Singh and Agarwal 1969) the contribution of the rescattering box diagram to the amplitude for the reactions which are forbidden in single-meson exchange. In performing these calculations, we have assumed that the imaginary part of the amplitude gives the dominant contribution in the energy region from an incident meson momentum of 1·5 GeV/*c* to a momentum of 5 GeV/*c*. The purpose of this letter is to show the justification of the above assumption by evaluating the real part of the box diagram amplitude.

For the reaction

$$\mathrm{K}^- + \mathrm{p} \to \Sigma^- + \pi^+ \tag{1}$$

the absorptive part of the box diagram amplitude can be extracted by putting the s channel particles on the mass shell, and thus reaction (1) can be treated as a two-step process (Singh and Agarwal 1969):

$$\mathrm{K}^- + \mathrm{p} \to \rho^0 + \Lambda \to \Sigma^- + \pi^+. \tag{2}$$

We thus evaluate the s channel discontinuity. The T matrix is related to the invariant amplitudes for the reaction (1) as follows:

$$\bar{u}(p_2)Tu(p_1) = \bar{u}(p_2)(-A(s,t) + \mathrm{i}\gamma . QB(s,t))u(p_1) \tag{3}$$

where $Q = \frac{1}{2}(q_1 + q_2)$, and p_1, p_2, q_1, q_2 are the momenta of initial and final baryons and mesons, respectively. To separate the imaginary part of the box diagram amplitude in terms of Im A and Im B, we use (Kubis and Gammel 1968),

$$\gamma_0 = \frac{\gamma_0(p_{10} + q_{10})}{\sqrt{s}}. \tag{4}$$

Here p_{10}, and q_{10} are the energies of the initial proton and kaon, respectively in the s channel centre-of-mass system, as s is the square of the total centre-of-mass energy. From equation (4), it is easy to get,

$$\bar{u}(p_2)\gamma_0 u(p_1) = -\bar{u}(p_2)[\tfrac{1}{2}(m_\mathrm{p} + m_\Sigma) + \mathrm{i}\gamma . Q]u(p_1)/\sqrt{s}. \tag{5}$$

Using (3) and (5), we can easily write the T matrix for the process (2) in terms of Im A

and Im B as follows:

$$\mathrm{Im}A(s,t) = \frac{Y|q'|}{4\pi\sqrt{s}} g_{\Lambda\pi\Sigma} g_{\rho\pi\pi} g_{\rho\mathrm{K}\bar{\mathrm{K}}} g_{\bar{\mathrm{K}}\mathrm{p}\Lambda} \left[m\frac{|q'|}{|q_1|} + \frac{m_\mathrm{p}+m_\Sigma}{2\sqrt{s}} \left(p_0' - p_{10}\frac{|q'|}{|q_1|} \right) + m_\Lambda \right]$$
$$\times \left[q_2 \cdot q_1 + \left(q_2 \cdot q_1 \frac{|q'|}{|q_1|} - q_{20} q_0' \right) \frac{|q_1||q'| - q_{10} q_0'}{m_\rho^2} \right],$$
$$\mathrm{Im}B(s,t) = -\frac{Y|q'|}{4\pi s} g_{\Lambda\pi\Sigma} g_{\rho\pi\pi} g_{\rho\mathrm{K}\bar{\mathrm{K}}} g_{\bar{\mathrm{K}}\mathrm{p}\Lambda} \left(p_0' - p_{10}\frac{|q'|}{|q_1|} \right)$$
$$\times \left[q_2 \cdot q_1 + \left(q_2 \cdot q_1 \frac{|q'|}{|q_1|} - q_{20} q_0' \right) \frac{|q_1||q'| - q_{10} q_0'}{m_\rho^2} \right] \tag{6}$$

with

$$Y = \frac{1}{4|q_1||q'|^2|q_2|} \frac{1}{\sqrt{-\beta_1}} \ln \frac{\alpha_1\alpha_2 - \cos\theta + \sqrt{-\beta_1}}{\alpha_1\alpha_2 - \cos\theta - \sqrt{-\beta_1}},$$
$$\beta_1 = 1 - \cos^2\theta - \alpha_1^2 - \alpha_2^2 + 2\alpha_1\alpha_2\cos\theta,$$
$$\alpha_1 = \frac{2q_{10}q_0' - m_\rho^2}{2|q_1||q'|},$$
$$\alpha_2 = \frac{2q_{20}q_2' - m_\rho^2}{2|q_2||q'|},$$

where $|q_1|$, $|q'|$ and $|q_2|$ are the magnitudes of the centre-of-mass momentum of the initial, intermediate and final mesons, respectively. Also p_0' (q_0') is the centre-of-mass energy of the intermediate $\Lambda(\rho^0)$ particle, and p_{20} (q_{20}) is the centre-of-mass energy of the final $\Sigma(\pi)$ particle.

Dispersive parts of the amplitudes (6) can be calculated by using the unsubtracted dispersion relations for the invariant amplitudes A and B, separately in the forward direction as follows:

$$\mathrm{Re}A(s) = \frac{p}{\pi}\int \frac{\mathrm{Im}A(s')}{s'-s} \mathrm{d}s',$$
$$\mathrm{Re}B(s) = \frac{p}{\pi}\int \frac{\mathrm{Im}B(s')}{s'-s} \mathrm{d}s'. \tag{7}$$

These integrals are undoubtedly convergent as the asymptotic behaviours of the amplitudes are:

$$\mathrm{Im}A(s') \sim \frac{1}{s'}$$
$$\mathrm{Im}B(s') \sim \frac{1}{s'^2}.$$

We have evaluated the numerical value of the integral by applying an arbitrary upper cut-off at 400 $(\mathrm{GeV})^2$.

In figure 1, we have plotted $\alpha(= \mathrm{Re}\, A/\mathrm{Im}\, A)$ and $\beta\ (= \mathrm{Re}\, B/\mathrm{Im}\, B)$ as a function of the incident kaon momenta. The results thus clearly indicate that the real parts of the box diagram amplitudes are smaller in magnitude than the imaginary parts in the

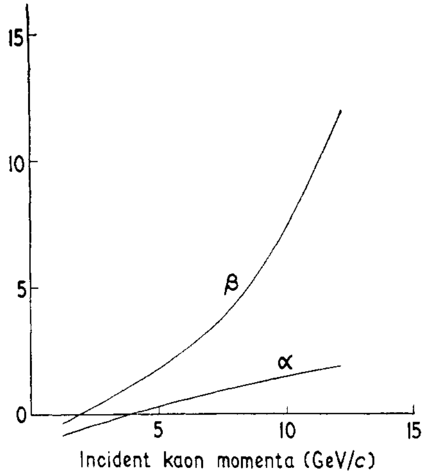

Figure 1. $\alpha(= \mathrm{Re}\, A/\mathrm{Im}\, A)$ and $\beta(= \mathrm{Re}\, B/\mathrm{Im}\, B)$ are plotted against the incident kaon momenta.

energy region from a kaon momentum of 1·5 GeV/c to a momentum of 4 GeV/c and, therefore, can be safely ignored while calculating the amplitude of the box diagram.

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